Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

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Mid Term Exam#1, Fall 2013

Name and SR.No:

Instructions:

- This is an open book, open notes exam. No wireless allowed.
- The time duration is 3 hrs.
- There are five main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and effort.
- Do not panic, do not cheat.
- Good luck!

Question No.	Points scored
1	
2	
3	
4	
5	
Total points	

PROBLEM 1: Examine if the following statements are true or false with correct reasoning. Random guessing or incorrect reasoning fetches zero credit. A statement is true if it is generic for all cases. A counter example is enough to make it false. All sub-parts of this problem carry equal credit.

(1) Alice and Bob play a game. Each one chooses a random number uniformly within the interval $[0, 1]$. The probability that sum of the numbers equals one is zero.

(2) The input to an LTI filter is a WSS random process. The output is always a WSS process.

(3) The signals $\{\cos(t), \cos(2t)\}$ over $[0, 2\pi]$ are linearly dependent.

(4) A causal LTI system is cascaded with another non-causal LTI. The overall system is always non-causal.

(20 pts.)

PROBLEM 2: This problem has two parts.

(1) A discrete LTI system takes an input $x[n]$ and yields $y[n]$. Obtain the necessary and sufficient conditions for the impulse response $h[n]$ so that $\max\{|x[n]|\} \geq \max\{|y[n]|\}$. (10 pts.)

(2) Let $(A_1, \mathbf{b}_1, \mathbf{c}_1^T)$ and $(A_2, \mathbf{b}_2, \mathbf{c}_2^T)$ denote two systems in state space representation. Obtain the overall system parameters $(A, \mathbf{b}, \mathbf{c}^T)$ when the two systems are connected in (a) parallel (b) series. (10 pts.)

PROBLEM 3: Consider a binary symmetric channel that we discussed in the class. The source sends '1' with probability p and '0' with probability $1 - p$. Due to noise, each bit is received incorrectly with a cross over probability ϵ.

(1) If we transmit the same information bit n times over the channel such that each transmission is statistically independent, determine the probability p_n that the information bit is '0' given that we observed a string of n zeros at the output. (7 pts.)

(2) Determine $\lim_{n\to\infty} p_n$. Interpret your solution. (7 pts.)

(3) Suppose an unknown bit is transmitted and the received bit is a '0'. Suppose the 'same' unknown bit is retransmitted again and we receive a '0'. What is the conditional probability that the second bit we received is a '0' given that the first bit received is a '0'? (6 pts.)

PROBLEM 4: This problem has 2 parts.

(1) If $\mathcal{W}_1$ and $\mathcal{W}_2$ are subspaces of a vector space $\mathcal{V}$, show that $\mathcal{W}_1 \cup \mathcal{W}_2$ is a subspace iff $\mathcal{W}_1 \subset \mathcal{W}_2$ or $\mathcal{W}_2 \subset \mathcal{W}_1$. (12 pts.)

(2) Suppose $\{\alpha_i\}_{i=1}^n$ be distinct real numbers. Examine if the exponential functions $\{e^{\alpha_i t}\}_{i=1}^n$ are all linearly independent over the space of real numbers. (8 pts.)

PROBLEM 5: Consider the inner product space of signals defined over $[-1, 1]$.

(1) Show that the signals 1 and t are orthogonal. (2 pts.)

(2) Obtain the least squares approximation of the signal $s(t) = t^{\frac{1}{2n+1}}$ over the interval $[-1, 1]$ using an orthonormal basis for the subspace spanned by $\{1, t\}$. Show all your steps. (18 pts.)