Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

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Mid Term Exam#2, Fall 2013

Name and SR.No:

Instructions:

- This is an open notes exam. Four A4 pages of written material on both sides is allowed. No wireless is allowed.
- The time duration is 3 hrs.
- There are four main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and effort for partial credit.
- Do not panic, do not cheat.
- Good luck!

Question No.	Points scored
1	
2	
3	
4	
Total points	

PROBLEM 1: This problem has 2 parts.

(1) Simplify the multirate systems shown in Figure 1 as best as you can. Obtain the frequency response $Y(z)$ in terms of $X(z)$. (15 pts.)

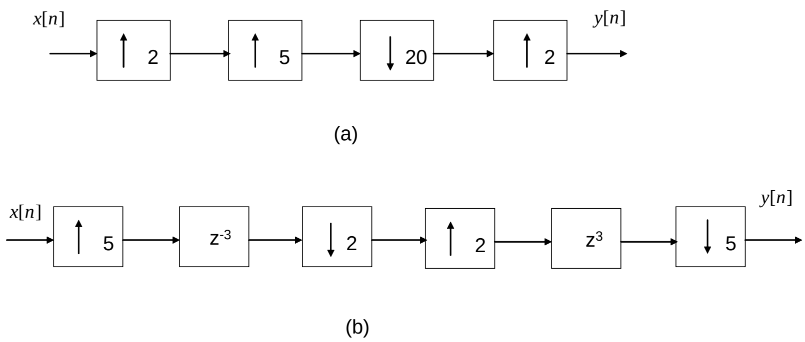

FIGURE 1. Two different multirate systems are shown in (a) and (b).

(2) Consider two systems shown in Figures 2 (a) and (b). Let k be some integer. Prove that the two systems are equivalent i.e., $y_0[n] = y_1[n]$ when $h_k[n] = h_0[n]\cos\left[\frac{2\pi kn}{L}\right]$. This is a structure where filtering followed by cosine modulation has the same effect as filtering with cosine modulated impulse response. Suppose $L = 5$ and $k = 1$. Let $X(e^{jw})$ and $H(e^{jw})$ be sketched as in Figure 2 (c). Sketch $Y(e^{jw})$, $Y_0(e^{jw})$ and $U(e^{jw})$. (15 pts.)

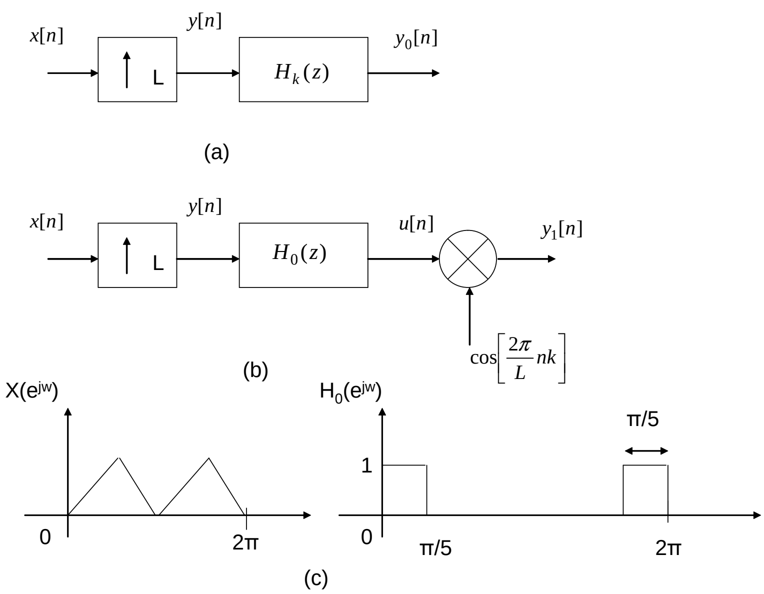

FIGURE 2. Two different interpolation systems are shown in (a) and (b). The input and base filter responses are shown as well in (c).

Hint: The Fourier transform for $e^{j\omega_0 n}$ is $2\pi\delta(\omega - \omega_0)$ for $0 < \omega < 2\pi$.

PROBLEM 2: This problem has two parts.

(1) Examine if the system shown in Figure 3 is a perfect reconstruction system. Show all your steps clearly from first principles. (20 pts.)

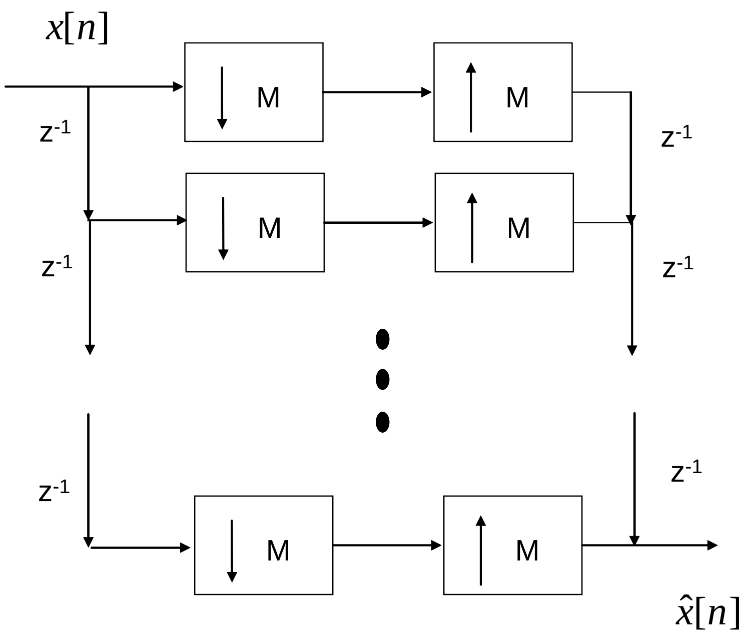

FIGURE 3. Delay filter bank.

(2) It is desired to build a circuit that works twice as fast for filtering downsampled signals through the filter $H(z) = \frac{1}{a+bz^{-1}}$. How would you accomplish this? (5 pts.)

PROBLEM 3: Let A be a linear transformation of vector x i.e., $y = Ax$. Let C_x denote the covariance matrix of data vector x. In the class, we proved that the transform that decorrelates y corresponds to Φ^T where Φ is the unitary matrix of eigenvectors corresponding to C_x. Suppose we intend to transform the covariance of y to a scaled version of the identity matrix i.e., $C_y = \sigma^2 I$. Show that $A = \sigma\Lambda^{-\frac{1}{2}}\Phi^T$, where Λ is a diagonal matrix of eigenvalues of C_x. Interpret the significance of this result geometrically for a scatter plot of 2-D points lying within an ellipse described by $\frac{(x_1-\mu_1)^2}{a^2} + \frac{(x_2-\mu_2)^2}{b^2} = 1$. (20 pts.)

PROBLEM 4: We want to model a signal $x[n]$ using an all-pole model of the form $H(z) = \frac{b(0)}{1+z^{-N}\left[\sum_{k=1}^{p} a_p(k)z^{-k}\right]}$.

(1) From first principles, derive the normal equations that define the coefficients $a_p(k)$ that minimizes the Prony error $E_p = \sum_{n=0}^{\infty} |e(n)|^2$. (12 pts.)

(2) Derive the expression for minimum error. How do you set $b(0)$? (13 pts.)