Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

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Mid Term Exam#1, Fall 2014

Name and SR.No:

Instructions:

- This is an open book, open notes exam. No wireless allowed.
- The time duration is 3 hrs.
- There are four main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- Do not panic, do not cheat.
- Good luck!

Question No.	Points scored
1	
2	
3	
4	
Total points	

PROBLEM 1: Examine if the following statements are true or false with correct reasoning. Random guessing or incorrect reasoning fetches zero credit. A statement is true if it is generic for all cases. A counter example is enough to make it false. All sub-parts of this problem carry equal credit.

(1) Let λ and x be an eigen value and the corresponding eigen vector of a square matrix $A := [a_{ij}]$ respectively. Then, $|\lambda| \geq ||A||$. The norm is the Hilbert Schmidt norm i.e., $\sqrt{\sum_i \sum_j |a_{ij}|^2}$.

(2) Let X and Y be any two random variables related by $Y = \phi(X)$ where, $\phi(.)$ is a monotonically increasing function. The probability distribution functions of the random variables are related as $F_Y(\phi(x)) \geq F_X(x)$.

(3) The energy of a signal $s(t)$ defined over $[0, T]$ is $E = \int_0^T s^2(t)dt$. Let $\bar{s}$ be the signal space representation of $s(t)$ over an appropriate basis. Then, $E = ||s||$ where $||.||$ is the usual $\mathcal{L}_2$ norm.

(4) Allpass filters are always causal and stable.

(5) We need $2n$ modes and 2^n measurements to determine the frequencies from a noise free linear mixture having n sinusoids.

(25 pts.)

PROBLEM 2: This problem has 2 parts.

(1) Suppose $\{\phi_i\}_{i=1}^{n}$ be a set of orthonormal basis for a vector space $\mathcal{V}$. Prove that for any two vectors x and y belonging to $\mathcal{V}$, $\langle x, y \rangle = \sum_{i=1}^{n} \langle x, \phi_i \rangle \overline{\langle y, \phi_i \rangle}$. (7 pts.)

(2) A source emits two signals $s_1(t) = u(t) - 2u(t-1) + u(t-2)$ and $s_2(t) = 4u(t) - 4u(t-2)$ where, $u(t)$ is the standard unit step function with probabilities p and $1 - p$ respectively. Represent the signals as vectors in an appropriate coordinate system. Let us suppose that the signal vectors are to be transmitted over an AWGN channel $\mathcal{N}(0, \sigma^2)$ where the noise cloud acts independently on each of the signal coordinates. Obtain the optimal linear decision boundaries for this system. Evaluate the probability of error. (18 pts.)

PROBLEM 3: This problem has 2 parts.

(1) Consider a 2 channel QMF bank with analysis and synthesis filters. Suppose $H_0(z) = 2 + 6z^{-1} + z^{-2} + 5z^{-3} + z^{-5}$, and $H_1(z) = H_0(-z)$. Obtain a set of stable synthesis filters for perfect reconstruction. What can you comment on amplitude and phase distortion of this filter bank? (15 pts.)

(2) Consider a 3 channel filter bank. Suppose we downsample by 2 and upsample by 2 in first branch. In the subsequent 2 branches, suppose we downsample and upsample by 4. Assume that the pass band width of the analysis filters is compliant with the decimation rate that follows it. From first principles, obtain the expression for the output signal after synthesis. Investigate if we can get alias free reconstruction of the signal after synthesis. What are the necessary conditions for the same? (20 pts.)

PROBLEM 4: Consider the equiripple FIR design. Let the signal be bandlimited to ω (π corresponds to Nyquist on the scale). Suppose we are interested in 3 separate operations (1) downsampling by M, (2) upsampling by L, (3) rational sampling rate conversion $\frac{L}{M}$.

(1) How would you choose the pass band edge frequencies of your filter? (6 pts.)

(2) How would you choose the stop band edge frequencies of your filter when (a) no aliasing is allowed (b) aliasing is permitted in the transition region of the decimator or beyond the Nyquist point in the interpolator as relevant to the three operations mentioned in this problem? (9 pts.)