

Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan G. Srinivasa

Home Work #4, Fall 2014

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, $d = \#$ days late

Assigned date: Nov 1th 2014

Due date: Nov 14th 2014 in class

PROBLEM 1: Consider the function $f(t)$ within the interval $[-2, 2]$.

$$f(t) = \begin{cases} 3 & -2 \leq t < -1 \\ -4 & -1 \leq t < 0 \\ 2 & 0 \leq t < 1 \\ 1 & 1 \leq t < 2 \end{cases}$$

- (1) Obtain the Haar wavelet decomposition of the function $f(t)$.
- (2) Sketch the frequency response at various stages in the wavelet decomposition.
- (3) Suppose the last stage in the wavelet decomposition is nulled out, obtain the reconstructed function after inverse wavelet transform. What fraction of the total signal energy is lost in this approximate reconstruction process?
- (4) Suppose we allocate only 3 bits for representing each sample after wavelet transform, obtain the reconstructed signal and compute the reconstruction error due to quantization.

(25 pts.)

PROBLEM 2: Six data points are given. They are $(-1, 1)^T$, $(0, 1)^T$, $(-3, -3)^T$, $(1, 0)^T$, $(1, -1)^T$ and $(3, 3)^T$ and occur with probabilities $1/4$, $1/16$, $3/16$, $3/16$, $1/16$, $1/4$ respectively.

- (1) Obtain the KL representation of these points.
- (2) Suppose we intend to retain only the dominant eigen component, what is the new representation of the points after dimensionality reduction? What fraction of signal energy is lost due to dimensionality reduction?
- (3) If the first three points correspond to class 1 and the next three points correspond to class 2, how many lines can separate the two classes? Determine the equation of a line so that the projection of the points on the line is minimized in the least squares sense for classification.
- (4) Suppose we intend to construct a line that separates the two classes after dimensionality reduction, determine the equation of the line so that the projection of the points on the line is minimized in the least squares sense.

(25 pts.)