Solutions Key

Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

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Mid Term Exam#1, Fall 2015

Name and SR.No:

Instructions:

- Only four A4 sheets of paper with written notes on both sides are allowed .
- The time duration is 3 hrs.
- There are five main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- Do not panic, do not cheat.
- Good luck!

Question No.	Points scored
1	
2	
3	
4	
5	
Total points	

PROBLEM 1: Examine if the following statements are true or false with correct reasoning. Random guessing or incorrect reasoning fetches zero credit. A statement is true if it is generic for all cases. A counter example is enough to make it false. All sub-parts of this problem carry equal credit.

(1) Let $\bar{v}$ be the eigenvector for a square matrix A with a corresponding eigenvalue zero. $\bar{v}$ is also an eigenvector for A^n for integers $n > 1$.

(2) Consider the random process $s(t) = A\sin(\omega t)$, where A is Gaussian distributed with mean zero and variance σ^2. The frequency ω is deterministic. $s(t)$ is a stationary process.

(3) Upsampling of a non-zero discrete time signal by a factor $L > 1$ always causes images within the base band.

(4) Consider a discrete time signal $x[n]$ at D samples/s coded at c bits/sample with energy predominantly in the low pass region. Suppose we pass this signal through a two channel QMF bank so that the output of the analysis bank is coded at a bits and b bits per sample in the low and high frequency subbands respectively during transmission. We can achieve a compression in the data rate if $2c > a + b$.

(5) If an LTI system is causal, it is always stable.

(20 pts.)

1) $A\bar{v} = 0 \cdot \bar{v}$ $(\lambda = 0)$ From eigenvalue eqn

Try $A^2\bar{v} = A \cdot A\bar{v} = A \cdot \vec{0} = 0$

Continuing, $A^n\bar{v} = \bar{0} \Rightarrow$ $\bar{v}$ is also an eigenvector for A^n $n > 1$.

$(\because A^{n-1} A v = 0 \cdot \bar{v})$

TRUE

2) Consider the 3rd moment and higher ... (odd ones $\neq 0$, even $\neq 0$)

$E(s^4(t)) = E(A^4)\sin^4(\omega t)$

$E(A^4) \neq 0$ for Gaussian $\Rightarrow$ 4th order statistics is a function of 't'

$\Rightarrow$ Not strict sense stationary.

FALSE

3) DFT is periodic with $2\pi \Rightarrow$ $\uparrow$ by L causes those spectra beyond base band to be 'wrapped' in due to compression

$\therefore$ Statement is true

4) Bit rate originally is Dc bits/s

At the subbands, from 2-channel QMF, bit rate $= \frac{D}{2}a + \frac{D}{2}b$ (due to downsampling by 2)

$\Rightarrow$ Compression rate (CR) $= \frac{Dc}{\frac{D}{2}(a+b)}$ $(CR > 1 \Rightarrow 2c > a+b)$

TRUE

5) Suppose $h[n] = u[n]$ (Causal)

$\sum_{n \geq 0} h(n) = \infty$ (Not stable)

FALSE