Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan G. Srinivasa

Home Work #1, Fall 2015

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Aug 30$^{\text{th}}$ 2015 **Due date:** Sep 9$^{\text{th}}$ 2015 in class

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PROBLEM 1: Provide an example of a 2D FIR filter with the following impulse response properties (a) non-causal (b) causal but unstable. Repeat this for the 2D IIR case. Justify your steps. (5 pts.)

PROBLEM 2: Problem 1.4.19 from the text Moon and Stirling (5 pts.)

PROBLEM 3: Problem 1.4.30 from the text Moon and Stirling. (5 pts.)

PROBLEM 4: Random variables X and Y are uniformly distributed in the interval $[0, 1]$. Assuming that X and Y are statistically independent, find the probability density function and the probability distribution function of a random variable $Z = |X - Y|$. (5 pts.)

PROBLEM 5: The power spectral density of a certain sequence $x[n]$ is $\frac{1}{a+b\cos(\omega)}$ for some non zero real constants a and b. Find the autocorrelation function. Suppose the autocorrelation function of a sequence $x[n]$ behaves as $r_{xx}(k) = \frac{1}{k}$ for time lags $k > 1$. What can you say on the power spectral density? (6 pts.)

PROBLEM 6: Suppose we are filtering a random sequence $x[n]$ through a FIR filter $1 - az^{-1}$, $|a| < 1$. Let $x[n]$ be a Bernoulli process such that $P(x[n] = 1) = p$ and $P(x[n] = 0) = 1 - p$. Examine if this is a wide sense stationary process and ergodic in the mean. (7 pts.)

PROBLEM 7: Consider a collection of all $n \times n$ matrices with real entries i.e., $M_n(R)$. Is this a vector space? Justify.

(1) Suppose we consider $\mathcal{S} := \{\mathbf{X} \in M_n(R) : \det(\mathbf{X}) = 0\}$, examine if $\mathcal{S}$ is a subspace.

(2) Suppose $\mathbf{P}, \mathbf{X} \in M_n(R)$. Let T be an operator such that $T(\mathbf{X}) = \mathbf{P}^T\mathbf{X}\mathbf{P}$ for a fixed matrix $\mathbf{P}$. Examine if T is linear.

(7 pts.)

PROBLEM 8: Consider the continuous time signal $f(t) = \sum_{i=0}^{N-1} a_i u(t - i\frac{T}{8})$ where a_i is any real number, T is a time unit and N is a positive integer. Let $\phi_1(t) = u(t) - u(t - \frac{T}{4})$ and $\phi_2(t) = u(t) - 2u(t - \frac{T}{8}) + u(t - \frac{T}{4})$.

(1) Are $\phi_1(t)$ and $\phi_2(t)$ linearly independent?

(2) Expand the signal $f(t)$ in the $\phi_1(t)$-$\phi_2(t)$ plane after normalizing $\phi_1(t)$ and $\phi_2(t)$. Interpret your results graphically.

(3) Suppose a source emits $\phi_1(t)$ and $\phi_2(t)$ randomly with source probabilities p and $1-p$ respectively. Imagine a 2D cloud of uncorrelated Gaussian noise $\mathcal{N}(\mathbf{0}, \sigma^{\mathbf{2}})$ acting in the $\phi_1(t)$-$\phi_2(t)$ plane. Determine the optimal linear decision boundaries to minimize the probability of misclassifying the signals $\phi_1(t)$ and $\phi_2(t)$. Explicitly evaluate the probability of misclassification.

(10 pts.)