# Indian Institute of Science

E9: 252: Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan Srinivasa Garani

Mid Term Exam#1, Fall 2017

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**Name and SR.No:**

**Instructions:**

- You are allowed only 5 pages of A4 pages written on both sides and a calculator for this exam. No wireless allowed.
- The time duration is 3 hrs.
- There are five main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- Make any reasonable assumptions if really required.
- Do not panic, do not cheat.
- Good luck!

| Question No. | Points scored |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| Total points | |

PROBLEM 1: This problem has 2 parts.

(1) Is the set $1, t, t^2, ..., t^m$ linearly dependent? Justify. (10) pts.

(2) Let $X = L_2[-\pi, \pi]$. Let $S_1 = \text{span}(1, \cos(t), \cos(2t), ...)$ and $S_2 = \text{span}(\sin(t), \sin(2t), ...)$. Examine if $\dim(S_1 \oplus S_2) = \dim(S_1) + \dim(S_2)$. (10 pts.)

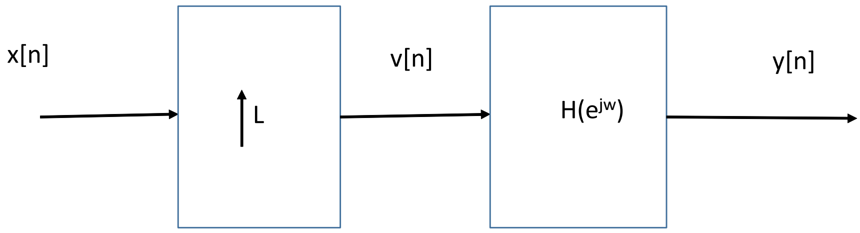
PROBLEM 2: This problem has 2 parts.

(1) Let $e[n]$ denote a white noise sequence, and let $s[n]$ be a sequence uncorrelated with $e[n]$. Examine if $y[n] = s[n]e[n]$ is white. (10 pts.)

(2) Let $x[n]$ be a real stationary white noise sequence with zero mean and variance $\sigma_x^2$. Determine the output variance if $x[n]$ is filtered through a cascade of two filters with responses $h_1[n]$ and $h_2[n]$. You can assume that the filters have infinite taps. (10 pts.)

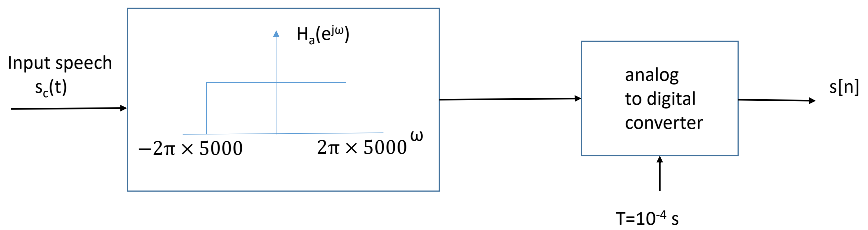
PROBLEM 3: Derive a general form of state space representation for $N$ cascaded LTI systems. Assume that each system in the cascade has a state space representation $\mathbf{A}_i, \mathbf{b}_i, \mathbf{c}_i^T, d_i = 0$ for $0 \leq i \leq N-1$. (10 pts.)

PROBLEM 4: The system shown in Figure approximately interpolates a discrete time sequence $x[n]$ by a factor $L$. Suppose that the linear filter has impulse response $h[n] = h[-n]$ and $h[n] = 0$ for $|n| > (RL-1)$, where $R$ and $L$ are integers; i.e., the impulse response is symmetric and of length $2RL - 1$ samples.

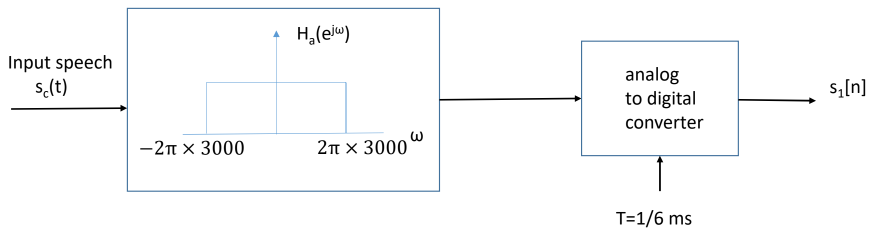



(1) How much delay must be inserted to make the system causal? (5 pts.)
(2) What conditions must be satisfied by $h[n]$ such that $y[n] = x[\frac{n}{L}]$ for $n = 0, \pm L, \pm 2L, ...$? (5 pts.)
(3) By exploiting the symmetry of the impulse response of the filter, show that each sample of $y[n]$ can be computed with no more than $RL$ multiplications. (5 pts.)
(4) By taking advantage of the fact that multiplications by zero need not be done, show that only $2R$ multiplications per output sample are required. (5 pts.)

PROBLEM 5: Suppose you obtained a sequence $s[n]$ by filtering a speech signal $s_c(t)$ with a continuous time low pass filter with a cutoff of 5 KHz and then sampling it at 10 KHz rate shown in Figure (a). Unfortunately, the speech signal $s_c(t)$ is destroyed once $s[n]$ was stored on a disk drive. Later you decided that you should have followed the process in Figure (b). Develop a method to obtain $s_1[n]$ from $s[n]$ using appropriate processing. Suppose it was required to filter $s_1[n]$ through a discrete time filter $H(z)$ for any post processing. Show how you will realize this efficiently using signals $s[n]$ and $H(z)$. (30 pts.)




(a)




(b)