Indian Institute of Science

E9−252: Mathematical Methods and Techniques in Signal Processing

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Homework #2, Fall 2017

Late submission policy: Points scored = Correct points scored $\times$ e^{-d}, d = # days late

Assigned date: Sept. 4$^{\text{th}}$ 2017 **Due date**: Sept. 11$^{\text{th}}$ 2017 by end of the day

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PROBLEM 1:

If $x(t) = \sum_{k=1}^{M} A_k e^{j2\pi f_k t}$, $E[A_k] = 0$ and A_k's are uncorrelated, examine if $x(t)$ is WSS.

PROBLEM 2:

Prove the following:

a) $|R_{XX}(\tau)| \leq R_{XX}(0)$

b) $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0)R_{YY}(0)}$

c) $R_{XX}(\tau) = R^*_{XX}(-\tau)$

d) $\sum_{k=1}^{N}\sum_{l=1}^{N} a_k a_l^* R_{XX}(t_k - t_l) \geq 0 \;\; \forall N > 0, \; \forall t_1 < t_2 < \cdots < t_N$ and complex a_i's

PROBLEM 3:

a) Only one of the switches S_1, S_2 and S_3 is active at a time. S_1 closes twice as fast as S_2. S_2 closes twice as fast as S_3.The signals are distributed normally as follows:

$$A \sim \mathcal{N}(-1, 4), B \sim \mathcal{N}(0, 1) \text{ and } C \sim \mathcal{N}(1, 4)$$

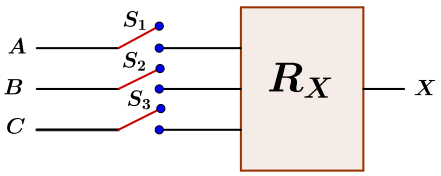

Figure 1: Feedback System

i) What is $P(X \leq 1)$?

ii) Given $X > -1$, which signal is most likely transmitted?

b) There are two roads from A to B and two roads from B to C. Each of the four roads have probability p of being blocked by snow independently of all the others. What is the probability of an open road from A to C?

PROBLEM 4: Prove the Cauchy Schwarz inequality for random variables: For two random variables X and Y,

$$|\text{Cov}(X, Y)| \leq \sqrt{\text{Var}(X)\text{Var}(Y)}.$$