PROBLEM 1:

Derive wavelet decomposition of a signal using m-adic Haar wavelets.

PROBLEM 2:

Let W_j be the space of all functions with basis $\psi(2^jt - k)$ where $k \in \mathbb{Z}$. Prove $V_{j+1} = V_j \oplus W_j$.

PROBLEM 3:

Obtain the Haar wavelet decomposition for the signal f(t) using the Haar basis. Indicate the signal dimension at each subspace. Sketch the waveforms explicitly at each subspace. Obtain the reconstructed signal in functional form after nulling out any spike of $(1/8)^{\text{th}}$ unit of time. Analyze using Fourier Transform. How much of energy is lost in the recovered signal?

$$f(t) = \begin{cases} 3 & 0 \le t < \frac{1}{4} \\ -1 & \frac{1}{4} \le t < \frac{3}{8} \\ 2 & \frac{3}{8} \le t < \frac{5}{8} \\ 0 & \frac{5}{8} \le t < 1 \end{cases}$$