NNSP-1

Homework #2

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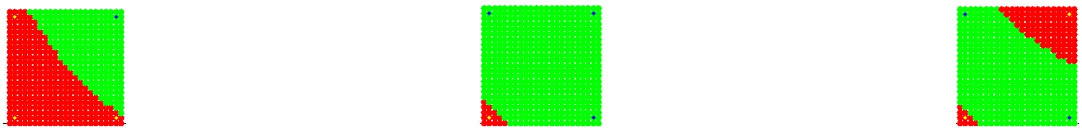
Problem 1

(a) The perceptron may be used to perform numerous logic functions. Demonstrate the implementation of the binary logic functions **AND**, **OR** and **COMPLEMENT**.

(b) A basic limitation of the perceptron is that it cannot implement the **EXCLUSIVE OR** function. Explain the reason for this limitation.

Solution



(a) AND. (b) OR. (c) XOR.

FIGURE 1 – Expected plots for Q1a,b

From Figure 1(c) we see that the data points are not linearly separable. A single perceptron will not be able to classify the two classes.

Problem 2

Consider one-dimensional data points that are Gaussian-distributed with means +1 and −1 and with a variance of 1. Design a classifier that separates these two classes. How would the classifier change if they have the same mean (say, 1) but different variances (say, 1 and 5) ? Generate the data points according to these probability distributions and comment upon the classification error.

Solution

From the class notes, for Baye's classifier, we have the following equation :

$$y = W^T X + b \tag{1}$$
$$y = \log(\Lambda(X)) \tag{2}$$
$$W = C^{-1}(\mu_1 - \mu_2) \tag{3}$$
$$b = \frac{1}{2}(\mu_2^T C^{-1}\mu_2 - \mu_1^T C^{-1}\mu_1). \tag{4}$$

From equation (1) we see that the data points are linearly separable. For $\mu_1 = \mu_2$ and $\sigma_1^2 \neq \sigma_2^2$ case, the decision boundary depends on the bias b in equation (1) .

Problem 3

It is required to classify the set of data points as shown in Figure 2 into two classes.

(a) Implement the multilayer perceptron (MLP) neural network architecture configured in online mode to classify the set of data points shown in blue and red in Figure 2. We expect you to show the decision boundary as well. Experiment and report the number of hidden layers and neurons minimally required for this classification task.

(b) Repeat part (a) using the MLP architecture configured in batch mode. Show the decision boundary as well.

Note : Choose an appropriate activation function, such as the tanh(.) function.

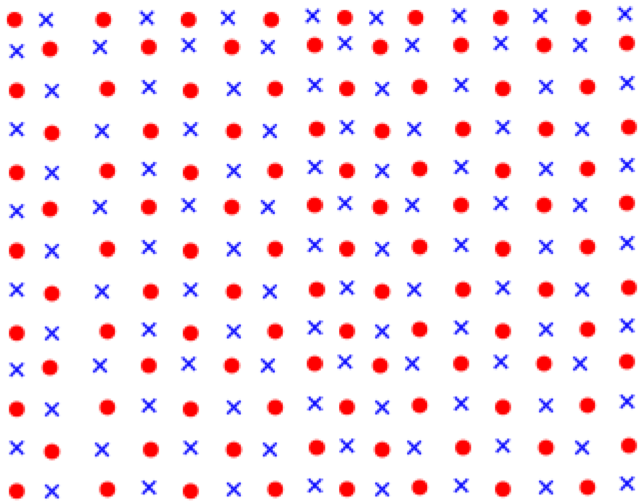


FIGURE 2 – Set of data points.

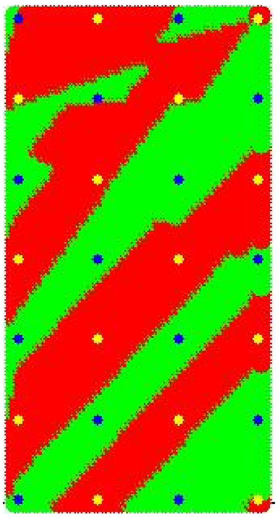


FIGURE 3 – Expected plot for Q3

Solution

The details of MLP architecture is given below :

1. Number of layers : 4
2. Number of hidden neurons : 10/layer
3. Learning rate : 0.15
4. Maximum number of epochs : 10000
5. Number of output neurons : 2

The expected output is shown in Figure 3. From Figure 3, we see that the set of data points are nothing but XOR data set repeated. Learning of decision boundaries i.e., (red) green strip in Figure 3 is just shifts of one of (red) green strip. This translates to learning the shift parameter keeping the weights constant.

Problem 4

Generate the set of data points as shown in Figure 4.

(a) Classify the set of data points into classes $\mathcal{C}_1$ and $\mathcal{C}_2$ using the perceptron algorithm configured in online and batch modes.

(b) Start with different initial conditions for the weight vector and the bias. Check whether you get the same decision boundary and comment upon this.

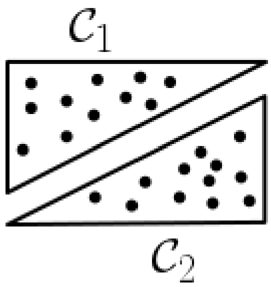
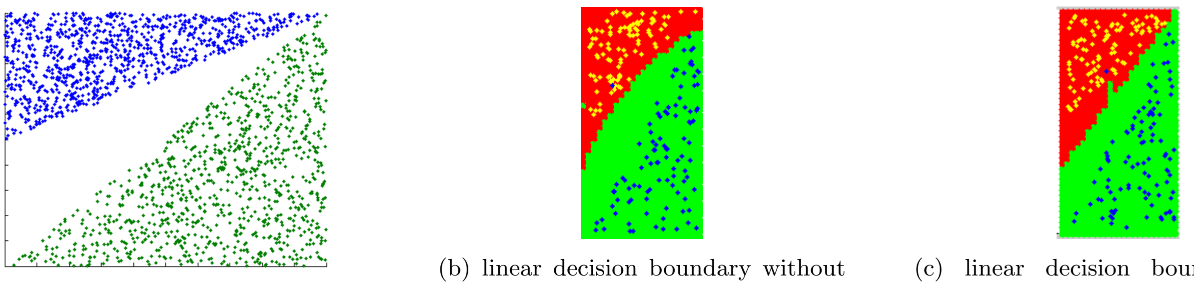


FIGURE 4 – Set of data points uniformly distributed within two right angled triangles forming classes $\mathcal{C}_1$ and $\mathcal{C}_2$.

(c) Add Gaussian noise with 0 mean and variance ranging from 0 to 5 to the set of data points shown in Figure 4. What is your stopping criterion for learning ? What can you comment upon the classification accuracy experimentally ?

Solution



(a) Data set.

(b) linear decision boundary without noise.

(c) linear decision boundary with noise.

FIGURE 5 – Expected plots for Q4