NNSP-1

Homework #5

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Solution for 8.5

Given : eigenvalue $\lambda_1 = 1 + \sigma^2$, corresponding eigenvector $q_1 = s$ and $R = ss^{\mathrm{T}} + \sigma^2\mathrm{I}$.
Consider

$$Rq_1 = ss^{\mathrm{T}}s + \sigma^2\mathrm{I}s \tag{1}$$
$$= \mathrm{I}s + \sigma^2\mathrm{I}s \tag{2}$$
$$= \left(\mathrm{I} + \sigma^2\mathrm{I}\right)s \tag{3}$$
$$= \lambda_1 q_1 \tag{4}$$

Solution for 8.15

Let us center $\overline{\Phi}(x_i)$ using the empirical mean as follows :

$$\overline{\Phi}(x_i) = \Phi(x_i) - \frac{1}{N}\sum_{i=1}^{N}\Phi(x_i). \tag{5}$$

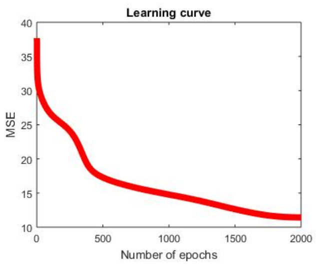
We define, $\overline{K}_{ij}$ as follows :

$$\overline{K}_{ij} = \langle\overline{\Phi}(x_i), \overline{\Phi}(x_j)\rangle. \tag{6}$$

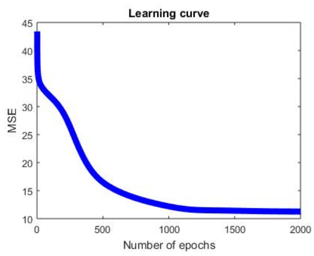
Substituting equation (5) in equation (6) and simplifying we get

$$\overline{K}_{ij} = K_{ij} - \frac{1}{N}\sum_{m=1}^{N}\Phi(x_m)^{\mathrm{T}}\Phi(x_j) - \frac{1}{N}\sum_{n=1}^{N}\Phi(x_i)^{\mathrm{T}}\Phi(x_n) + \frac{1}{N^2}\sum_{m=1}^{N}\sum_{n=1}^{N}\Phi(x_m)^{\mathrm{T}}\Phi(x_n) \tag{7}$$

Solution for 8.17



(a) Learning curve of GHA for Lena image.



(b) Learning curve of GHA for pepper image.

FIGURE 1 – Learning curves of GHA

Solution for 8.18

Given :

1. $x_1 \in \mathrm{U}[-1, 1]$.
2. $x_2 = x_1^2 + v$.
3. $v \in \mathcal{N}(0, 0.04)$.
4. Use Kernel Hebbian Algorithm.

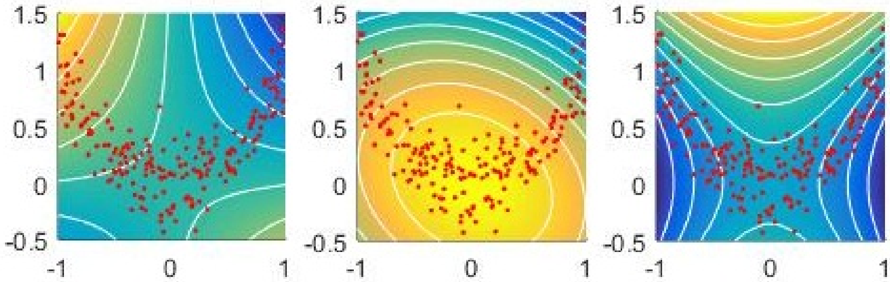


FIGURE 2 – Kernel Hebbian algorithm for 2D data.

1. Visually, Figure 2 is similar to the Figure 8.13 in Haykin book.