E9-251: Signal Processing for Data Recording Channels

Exam #2

Name:

Department:

Instructions:

- The paper contains 4 main questions. Attempt all the questions with careful reasoning and justifying your approach.
- Think carefully before you proceed and reason out your end results. The problems need more thought than calculations.
- The exam is of 3 hours duration.
- You can refer to any book/notes.
- Do not panic, do not cheat.
- Good Luck!

Question No	Max Points	Actual Score
1	25	
2	25	
3	20	
4	30	

Total:

1) The continuous time response of a Lorentzian channel in longitudinal recording is given by

$$h(t) = \frac{A}{1+\left(\frac{2t}{PW_{50}}\right)^2}; -\infty \le t \le \infty$$

. Let the symbol density be $D = 2$, and the system is sampled at baud rate. We are interested in designing a zero forcing equalizer for this system as shown in Figure 1, i.e., $F(z)H(z) = 1$.

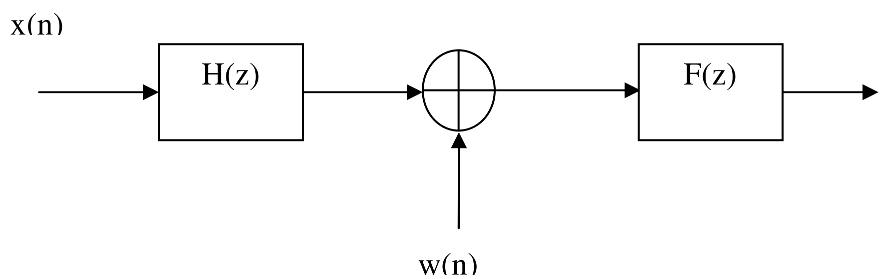

Figure 1: Schematic for a zero forcing equalizer

(a) Sketch $h(t)$ for $D = 2$ and baud rate sampling. (5 pts)

(b) Design a 3 tap zero forcing equalizer for this system. (12 pts)

(c) If noise $w(n)$ is Gaussian distributed $N(0, \sigma^2)$, what is the noise variance after zero forcing equalization? Is the noise power boosted? (8 pts)

(Hint: Sample $h(t)$ at times $t = kT, k = 0, \pm 1$ and appropriately solve for $F(z)$)

2) Consider a simplified discrete version of a data storage channel as shown in Figure 2. Random but equally likely NRZ data $a_k \in \{-1,1\}$ is stored on a recording channel. The read back signal forms a discrete sequence $s(n)$. The goal would be to design an FIR equalizer $F(z) = \sum_{i=0}^{K} f_i z^{-i}$ of order K that matches a <u>known</u> partial response target $P(z) = \sum_{i=0}^{L} p_i z^{-i}$ in the mean squared error (MSE sense).

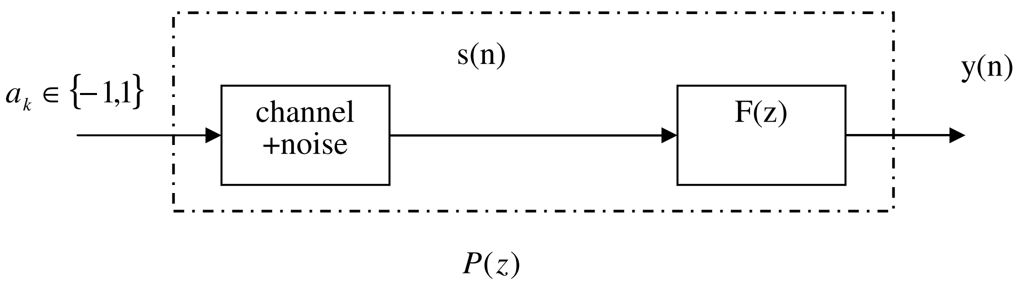

Figure 2: Schematic of a partial response equalizer

(a) Formulate an expression for the mean square error. (Hint: Use the Hessian matrix form of representation for computational ease). Indicate in your expression all the dimensions of the matrices involved carefully.

(10 pts)

(b) Prove that the optimal filter $\mathbf{F} = \mathbf{R}^{-1}\mathbf{QP}$ where $\mathbf{R} := [r_{ij}], r_{ij} = E(s_{n-i}s_{n-j})$ and $\mathbf{Q} := [q_{ij}], q_{ij} = E(s_{n-i}a_{n-j})$. (Hint: the length of the vector $\mathbf{F}$ must be exactly K+1).

(15 pts)

3) Consider a first order digital PLL shown in Figure 3.

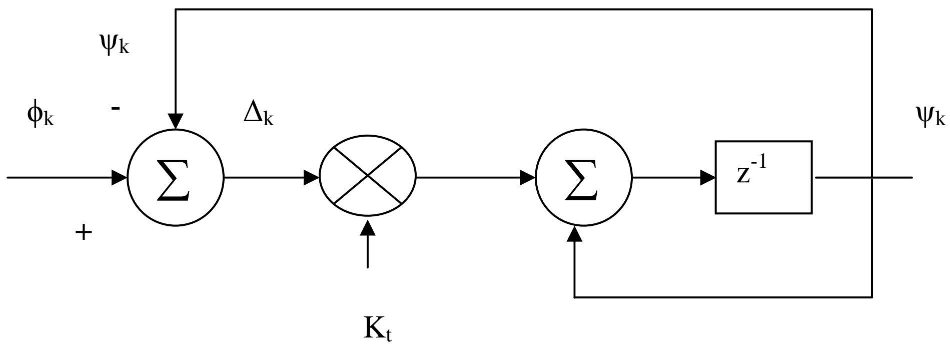

Figure 3: Schematic of a first order digital PLL

(a) For a unit step phase input $\phi_k = u(k)$, find the phase error. Under what conditions for K_t will the phase error exhibit oscillatory behavior. (10 pts)

(b) Repeat part (a) for a frequency jump at the phase input i.e., $\phi_k = \lambda k u(k)$. (10 pts)

4) A random signal x has a probability density $f_X(x)$. The signal is valid in the range $[0, L]$. It is desired to quantize this signal over K uniform intervals over $[0, L]$. Over the i^th^ interval $[a_{i-1}, a_i]$ we choose a quantized level y_i representing that interval.

(a) Show that the average mean square distortion due to quantization is given by

$$D = \sum_{i=1}^{K} \int_{a_{i-1}}^{a_i} (x - y_i)^2 f_X(x) dx \qquad \text{(10 pts)}$$

(b) Find the values of y_i that minimize the distortion function D. (15 pts)

(c) Suppose $f_X(x)$ is uniformly distributed over $[0, L]$ and we choose 2 equal sub-intervals for quantization. Using your intuition, how would you choose the two quantization levels that will minimize D? (2 pts)

(d) If $D = 0$, what should be the values of y_i and K? (3 pts)