Indian Institute of Science

E9 − 207: Basics of Signal Processing

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Homework #1, Spring 2018

Late submission policy: Points scored = Correct points scored $\times$ e^{-d}, d = # days late

Assigned date: Jan. 18$^{\text{th}}$ 2018 **Due date**: Feb. 1$^{\text{st}}$ 2018 in class

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PROBLEM 1:
Give examples (if any) of 2D discrete time systems that are (a) non-causal but stable (b) non-causal and unstable. (4 pts.)

PROBLEM 2:
Find the modes of a system whose impulse response is $y[n] = n^k u[n]$, where k is a positive integer. Write the governing difference equation for this system. (6 pts.)

PROBLEM 3:
Three coins C_1, C_2 and C_3 each show heads with probability 3/5 and tails otherwise. C_1 counts 10 points for a head and 2 for a tail, C_2 counts 4 points for both head and tail and C_3 counts 3 points for a head and 20 for a tail. You and your opponent each choose a coin; you cannot choose the same coin as your opponent. Each of you tosses your coin and the person with the larger score wins 10 dollars. Would you prefer to be the first to pick a coin or the second? Explain. (5 pts.)

PROBLEM 4:
Three companies A, B, C manufacture light bulbs and have a market share in the ratio $0.35 : 0.35 : 0.3$. Probability of each of them producing a defective bulb is respectively $0.01, 0.02$ and 0.05. A randomly chosen bulb is found defective. What is the probability that it was manufactured by company B? (4 pts.)

PROBLEM 5:
(a) If A and B are independent events. Show that A^c and B^c are independent. (3 pts.)

(b) If A and B are two events in Ω, show that (i)$(A \cup B)^c = A^c \cap B^c$ (ii)$(A \cap B)^c = A^c \cup B^c$ (4 pts.)

PROBLEM 6:
If X is a continous random variable with pdf given by $f(t) = \frac{1}{\pi(1+t^2)}$, for $-\infty < t < \infty$. What can you say about the mean and variance of X? (6 pts.)

PROBLEM 7:
Consider 2D points lying on a circular disk of radius R centered at origin.

$$S \coloneqq \left\{(x,\, y) :\, x^2 + y^2 \leq R^2\right\}$$

All points $(x, y) \in S$ are uniformly distributed on the disk. Obtain the marginal distributions $f_X(x)$ and $f_Y(y)$. Are they statistically independent? Are they correlated? (8 pts.)

PROBLEM 8:

Consider two uniformly distributed $U[0, 1]$ random variables X and Y. Let us form a new random variable $Z = |X - Y|$. Find and sketch (a) Probability distribution function of Z (b) Probability density function of Z. Also compute the mean and variance of Z. (10 pts.)