Assignment #00 - Solution Manual

1. Let $a(n)$ = [-1 1 -1 1] and $b(n)$ = [-1 0 2 3 4] be two discrete time signals. What is the signal obtained when a(n) is linearly convolved with b(n)? **1 point**

   - ■ [1 -1 -1 -2 -3 3 -1 4]
   - □ [-4 1 -3 3 2 1 1 -1]
   - □ [1 0 -2 3 0]
   - □ [-8 8 -8 8]

Solution:

Let the given two signals $a(n)$ and $b(n)$ of length l_1 and l_2 be linearly convolved to obtain signal $s(n) = a(n)*b(n)$, where $*$ depicts linear convolution. The length of $s(n) = l_1+l_2-1 = 4+5-1 = 8$. Due to this, we can eliminate the last two options. To compute the linear convolution, we use the tabular method,

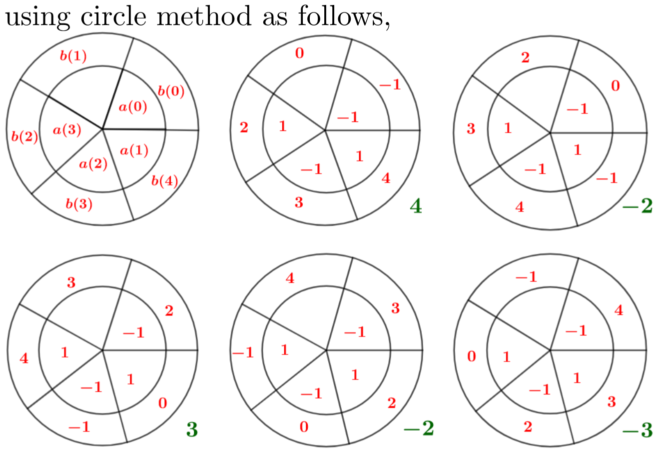
From the table, we obtain, $s(n) = [1\ -1\ -1\ -2\ -3\ 3\ -1\,4]$.

2. Let $a(n)$ = [-1 1 -1 1] and $b(n)$ = [-1 0 2 3 4] be two discrete time signals. What is the signal obtained when a(n) is circularly convolved with b(n)? **1 point**

   - ■ [4 -2 3 -2 -3]
   - □ [-3 2 -4 3 2]
   - □ [1 0 -2 3 0]
   - □ [-4 1 -3 3 2 1 1 -1]

Solution:

Let the given two signals $a(n)$ and $b(n)$ of length l_1 and l_2 be circularly convolved to obtain signal $s(n) = a(n) \circledast b(n)$, where $\circledast$ depicts circular convolution. The length of $s(n) = \max(l_1, l_2) = \max(4, 5) = 5$. Due to this, we can eliminate the last option. We compute the circular convolution using circle method as follows,



We multiply the elements in the corresponding circle sectors and add them up to obtain $s(n) = [4\ -2\,3\ -2\ -3]$.

3. Let $s(t) = 4\delta(t+4) + 3\delta(3t+2) + 4\delta(4t-1) + 5\delta(t-5)$. Let $y(t)$ be the output response to a system with transfer function $H(\mathrm{f}) = \mathrm{sinc}(\mathrm{f}) = \frac{\sin(\pi \mathrm{f})}{\pi \mathrm{f}}$ and input $s(t)$. Compute $\int\limits_{-\infty}^{\infty} Y(\mathrm{f})\mathrm{df}$, where $Y(\mathrm{f})$ is the Fourier transform of $y(t)$. **3 points**

   - ☐ 0
   - ■ **4**
   - ☐ 7
   - ☐ 16

Solution:

As $y(t)$ is the output response of a system, $y(t) = s(t) * h(t)$. Fourier transform (FT) of a signal $s(t)$ at frequency f is given by,

$$S(\mathrm{f}) = \int_{-\infty}^{\infty} s(t)\mathrm{e}^{-j2\pi ft}\mathrm{d}t.$$

We know that $H(\mathrm{f})$ is the Fourier transform of $h(t) = \mathrm{rect}_1(t) = \begin{cases} 1 & \frac{-1}{2} \leq t \leq \frac{1}{2} \\ 0 & \text{else} \end{cases}$. The inverse Fourier transform is computed as $y(t) = \int\limits_{-\infty}^{\infty} Y(\mathrm{f})\mathrm{e}^{j2\pi ft}\mathrm{df}$. Thus, the integral to be computed is equal to $y(0)$. $y(0) = \int\limits_{-\infty}^{\infty} s(\tau)h(0-\tau)\mathrm{d}\tau = \int\limits_{-1/2}^{1/2} 4\delta(-\tau+4) + 3\delta(-3\tau+2) + 4\delta(-4\tau-1) + 5\delta(-\tau-5)\mathrm{d}\tau = 4$ as $h(t)$ is a rectangular pulse between -1/2 and 1/2.

4. True or False : FIR filters are used widely because IIR filters need infinite memory and hence are hard to realize. <u>**False**</u> **1 point**

Solution:

Let $x(n)$ and $y(n)$ be the input and output of the system with transfer function $H(\mathrm{z})$ which is an IIR filter.

IIR filter transfer function takes the form $H(\mathrm{z}) = \frac{Y(\mathrm{z})}{X(\mathrm{z})} = \frac{\sum\limits_{i=0}^{p} b_i z^{-i}}{1+\sum\limits_{i=1}^{q} a_i z^{-i}}$, for finite p and q

$\Rightarrow (1 + \sum\limits_{i=1}^{q} a_i z^{-i})Y(\mathrm{z}) = \sum\limits_{i=0}^{p} b_i z^{-i} X(\mathrm{z}) \Rightarrow y(n) = \sum\limits_{i=0}^{p} b_i x(n-i) - \sum\limits_{i=1}^{q} a_i y(n-i)$.

As $y(n)$ is computed using the current input, q past outputs and p past inputs, hence we need only $p+q$ memories in the filter. Thus, the statement is false.

5. Based on the impulse response, choose FIR filters: **2 points**

   - ☐ $H(z) = \frac{1+z^{-1}}{1-z^{-1}}$
   - ■ $H(z) = \frac{1-z^{-2}}{1+z^{-1}}$
   - ■ $H(z) = \sum\limits_{i=0}^{25} a_0 z^{-i}$

■ $H(z) = \frac{1}{z}$

Solution:

FIR filter transfer function takes the form $H(z) = \sum_{i=0}^{p} b_i z^{-i}$.

$H(z) = \frac{1-z^{-2}}{1+z^{-1}} = 1 - z^{-1}$, $H(z) = \sum_{i=0}^{25} a_0 z^{-i}$ and $H(z) = \frac{1}{z} = z^{-1}$ are in this form and hence are FIR filters. $H(z) = \frac{1+z^{-1}}{1-z^{-1}}$ represents an IIR filter.

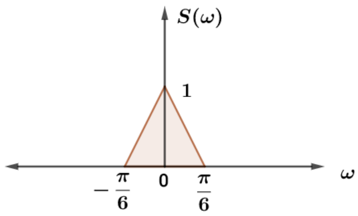
6. What is the DC component of the signal $s(t) = \frac{t}{1+t^2}$? **2 points**

- ☐ ∞
- ■ **0**
- ☐ 2
- ☐ 1

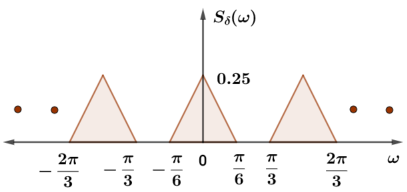
Solution:

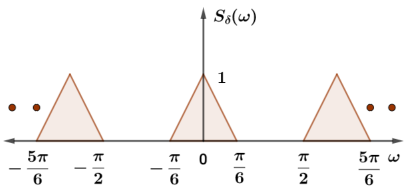
DC component of any signal is the amplitude of the component of the signal having frequency of 0 Hz. Let $S(f)$ be the Fourier transform of $s(t)$. Thus, the DC component of the signal is given by $S(0) = \int_{-\infty}^{\infty} s(t)dt = \int_{-\infty}^{\infty} \frac{t}{1+t^2}dt = 0$. The last equality follows from the fact that $s(t)$ is an odd signal.

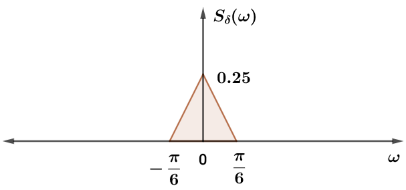
7. Let $s(t)$ be a bandlimited signal given in the figure below: **2 points**

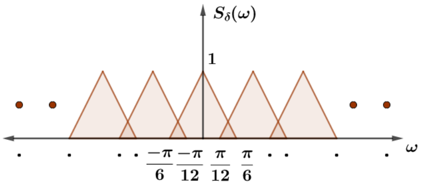


Which of the following figures represent the spectrum $S_\delta(\omega)$ of the signal obtained after sampling $s(t)$ at the rate $f_s = \frac{1}{4}$Hz where $\omega = 2\pi f$:

■ 

☐ 

☐ 

☐ 

> **Solution:**
>
> If the spectrum of $s(t)$ is $S(\mathrm{f})$, then the spectrum $S_\delta(\mathrm{f})$ of the sampled signal $s_\delta(t)$, sampled at a rate of f_s Hz is given by,
>
> $$S_\delta(\mathrm{f}) = f_s \sum_{m=-\infty}^{\infty} S(\mathrm{f} - mf_s).$$
>
> $S_\delta(\mathrm{f})$ is the scaled version of the sum of the frequency translates of $S(\mathrm{f})$. We call each translate as an image. Note that $f_s = \frac{1}{4} \Rightarrow \omega_s = 2\pi f_s = \frac{\pi}{2}$. We eliminate the 2nd and the last option as they have amplitude of 1, not 0.25. In $S_\delta(\mathrm{f})$, the frequency difference between the corresponding frequencies in 2 consecutive images must be $\omega_s = \frac{\pi}{2}$, which occurs in the first option. Thus, the first option depicts $S_\delta(\mathrm{f})$.

8. Let $H(z) = 1 + a^2z^{-2} + a^4z^{-4} + a^6z^{-6} \cdots$, $|a| < 1$ be the impulse response of a LTI system. Let $X(z) = 1 + az^{-1}$ be the input to the system and let $Y(z)$ be the corresponding output signal. What are the poles and zeros of the signal $Y(z)$? **1 point**

   - ☐ Poles: {a, -a}, Zeros :{-a}
   - ☐ Poles: {-a}, Zeros :{}
   - ■ **Poles: {a}, Zeros :{}**
   - ☐ Poles: {a}, Zeros :{-a}

> **Solution:**
>
> $H(z) = 1 + a^2z^{-2} + a^4z^{-4} + \cdots = \frac{1}{1-a^2z^{-2}} = \frac{1}{(1-az^{-1})(1+az^{-1})}$
>
> $Y(z) = H(z)\,X(z) = \frac{1}{(1-az^{-1})(1+az^{-1})} \times 1 + az^{-1} = \frac{1}{1-az^{-1}}$
>
> $Y(z)$ has a pole at $z = a$, and no zeros.

9. Let $Y(z) = a + z$ be the output of a LTI system when the input is $X(z) = 1 - az^{-1}$. True/False: The LTI system is causal. ____**False**____ **1 point**

> **Solution:**
>
> The output $Y(z) = a + z$ is
>
> $$y[n] = \begin{cases} 1 & n = -1, \\ a & n = 0, \\ 0 & \text{otherwise.} \end{cases}$$
>
> for the input
>
> $$x[n] = \begin{cases} 1 & n = 0, \\ -a & n = 1, \\ 0 & \text{otherwise.} \end{cases}$$
>
> Essentially, the output $y[n]$ begins at time $n = -1$, even before there is input to the system. Therefore the system is non-causal.

10. Let $Y(z) = a + z$ be the output of a LTI system when the input is $X(z) = 1 - az^{-1}$. What are the conditions under which the LTI system is BIBO stable? **1 point**

- ☐ Always
- ■ $|a| < 1$
- ☐ $|a| > 1$
- ■ $|a| < 0.5$

Solution:

Since the system is LTI, the transfer function is

$$H(z) = \frac{Y(z)}{X(z)} = \frac{a + z}{1 - az^{-1}}.$$

$H(z)$ has a pole at $z = a$. For the system to be stable, we require all poles to be within unit-circle on the z-plane. Therefore the system is stable as long as $|a| < 1$.

System is also stable if $|a| < 0.5$

11. Let $Y(z) = a + z$ be the output of a LTI system when the input is $X(z) = 1 - az^{-1}$. What is the impulse response of the system under the condition that the system is BIBO stable? **1 point**

- ■ $z + 2a + 2a^2z^{-1} + 2a^3z^{-2} + 2a^4z^{-3} \cdots$
- ☐ $z - 2a + 2a^2z^{-1} - 2a^3z^{-2} + 2a^4z^{-3} \cdots$
- ☐ $z + (1 - a) + (a - a^2)z^{-1} + (a^2 - a^3)z^{-2} + (a^3 - a^4)z^{-3} \cdots$
- ☐ $z + (1 + a) + (a + a^2)z^{-1} + (a^2 + a^3)z^{-2} + (a^3 + a^4)z^{-3} \cdots$

Solution:

The impulse response is

$$\begin{aligned} H(z) &= (a + z)\left(\frac{1}{1 - az^{-1}}\right) = (a + z)\left(1 + az^{-1} + a^2z^{-2} + a^3z^{-3} \cdots\right) \\ &= a\left(1 + az^{-1} + a^2z^{-2} + a^3z^{-3} \cdots\right) + \left(z + a + a^2z^{-1} + a^3z^{-2} \cdots\right) \\ H(z) &= z + 2a + 2a^2z^{-1} + 2a^3z^{-2} \cdots \end{aligned}$$

12. If $1, \omega, \omega^2, \cdots, \omega^{N-1}$ are the N$^{\text{th}}$ roots of unity, what is the value of summation $\sum_{i=0}^{N-1} \omega^{2i}$? **2 points**

- ■ **0 when N > 2; N otherwise.**
- ☐ 2 when N is even; 0 when N is odd.
- ☐ 2 when N is even; N when N is odd.
- ☐ 0 always.

Solution:

The answer can be obtained by verifying for $N = 1, 2, 3, 4$. Following proves the generic result.

We have $\omega = e^{j\frac{2\pi}{N}}$. For $N = 1$, the summation is trivially 1.

For $N = 2$, $\{1, -1\}$ are the roots. The summation is $1^2 + (-1)^2 = 2$.

For $N > 2$ and N even, we can write $N = 2M$.

$$\begin{aligned}\sum_{i=0}^{N-1}\omega^{2i} &= \sum_{i=0}^{N-1} e^{j\frac{2\pi\times 2i}{N}} = \sum_{i=0}^{N-1} e^{j\frac{2\pi i}{M}} = \sum_{i=0}^{M-1} e^{j\frac{2\pi i}{M}} + \sum_{i=M}^{2M-1} e^{j\frac{2\pi i}{M}} \\ &= \sum_{i=0}^{M-1} e^{j\frac{2\pi i}{M}} + \sum_{k=0}^{M-1} e^{j\frac{2\pi(k+M))}{M}} \quad (k = i - M) \\ &= \sum_{i=0}^{M-1} e^{j\frac{2\pi i}{M}} + \sum_{k=0}^{M-1} e^{j\frac{2\pi k}{M}} e^{j2\pi} = \sum_{i=0}^{M-1} e^{j\frac{2\pi i}{M}} + \sum_{k=0}^{M-1} e^{j\frac{2\pi k}{M}}\end{aligned}$$

$\sum_{i=0}^{M-1} e^{j\frac{2\pi i}{M}}$ is the sum of all M^{th} roots of unity for $M > 1$, which is zero. Hence, $\sum_{i=0}^{N-1}\omega^{2i} = 0$.

For $N > 2$ and N odd, we can write $N = 2M + 1$.

$$\begin{aligned}\sum_{i=0}^{N-1}\omega^{2i} &= \sum_{i=0}^{N-1} e^{j\frac{2\pi\times 2i}{N}} = \sum_{i=0}^{M} e^{j\frac{4\pi i}{2M+1}} + \sum_{i=M+1}^{2M} e^{j\frac{4\pi i}{2M+1}} \\ &= \sum_{i=0}^{M} e^{j\frac{4\pi i}{2M+1}} + \sum_{k=0}^{M-1} e^{j\frac{4\pi(k+M+1)}{2M+1}} \quad (k = i - M - 1) \\ &= \sum_{i=0}^{M} e^{j\frac{4\pi i}{2M+1}} + \sum_{k=0}^{M-1} e^{j\frac{2\pi((2k+1)+(2M+1))}{2M+1}} = \sum_{i=0}^{M} e^{j\frac{4\pi i}{2M+1}} + \sum_{k=0}^{M-1} e^{j\frac{2\pi(2k+1)}{2M+1}} e^{j2\pi} \\ &= \sum_{i=0}^{M} e^{j\frac{2\pi}{2M+1}\times 2i} + \sum_{k=0}^{M-1} e^{j\frac{2\pi}{2M+1}(2k+1)} \\ &= \underbrace{\sum_{i=0}^{M}\omega^{2i}}_{\text{all even powers of } \omega} + \underbrace{\sum_{k=0}^{M-1}\omega^{2k+1}}_{\text{all odd powers of } \omega} = \sum_{i=0}^{2M}\omega^{i} = 0.\end{aligned}$$

Therefore,

$$\sum_{i=0}^{N-1}\omega^{2i} = \begin{cases} 1 & N = 1, \\ 2 & N = 2, \\ 0 & \text{otherwise} \end{cases}$$

where ω is the N^{th} root of unity.

13. What is the energy in the signal $x(t) = \frac{\sin(2\pi t)}{\pi t}$? (Hint: Parseval's theorem) **2** **2 points**

Solution:

Fourier transform of $x(t) = \frac{\sin(2\pi t)}{\pi t}$ is given by

$$X(f) = \begin{cases} 1, & -1 \le f \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Using Parseval's theorem, energy in the signal

$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |X(f)|^2 \, df = \int_{-1}^{1} 1 df = 2.$$

14. Consider the following matrix **3 points**

$$A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$$

If the eigenvalues of A are 4 and 8, then

- ■ $x = -4, \; y = 10$
- ☐ $x = 5, \; y = 8$
- ☐ $x = -3, \; y = 9$
- ☐ $x = 4, \; y = 10$

Solution:

We know that the sum of eigenvalues is equal to the trace and the product is equal to the determinant of the matrix. Using these properties, we get $4 + 8 = 2 + y \implies y = 10$ and $4 \times 8 = 2y - 3x \implies 32 = 20 - 3x \implies 12 = -3x \implies x = -4$. Hence, $x = -4$ and $y = 10$.

15. Let $\mathbf{x} = [1, 2, \ldots, 100]^{\mathrm{T}}$ be a column vector of size 100×1. Then the rank of the matrix $\mathbf{x}\mathbf{x}^{\mathbf{T}}$ is **3 points**

- ☐ 99
- ☐ 100
- ■ **1**
- ☐ 0

Solution:

All the rows of the matrix $\mathbf{x}\mathbf{x}^{\mathbf{T}}$ are linearly dependent on the $1st$ row. Therefore the rank is 1.

16. The determinant of the matrix $\begin{bmatrix} -1 & 3 & 2 & 8 \\ 0 & 2 & 8 & 1 \\ 0 & 0 & 6 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ is **2 points**

- ☐ 11
- ☐ 48
- ■ –48
- ☐ 0

Solution:

For an upper triangular matrix, the entries on the diagonal are the eigenvalues. Using the property that the product of eigenvalues is the determinant, we get –48.

17. Let A and B be two matrices of size $n \times n$ such that $\det(AB) = 0$ and $\det(A) \neq 0$, then **3 points**

- ☐ all the eigenvalues of B must be zero.
- ☐ none of the eigenvalues of B must be zero.
- ■ **at least one of the eigenvalues of B must be zero.**
- ☐ B should be a zero matrix.

Solution:
We know that $\det(AB) = \det(A)\det(B)$ and $\det(AB) = 0 \implies \det(A)\det(B) = 0$. It is given that $\det(A) \neq 0$. Therefore, $\det(B) = 0$. For this to happen, at least one of the eigenvalue of B should be zero as we know that the product of eigenvalues is the determinant.

18. If A is a real square matrix, then AA^{T} is **2 points**

- ☐ unsymmetric
- ☐ sometimes symmetric
- ■ **always symmetric**
- ☐ skew-symmetric

Solution:
$(AA^{\mathrm{T}})^{\mathrm{T}} = (A^{\mathrm{T}})^{\mathrm{T}}A^{\mathrm{T}} = AA^{\mathrm{T}}$.
Therefore, it is always symmetric.

19. Let $A = \begin{bmatrix} 5 & 3 \\ 10 & p \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$. $Ax = b$ has a unique solution when $p \neq$ ____**6**____. **2 points**

Solution:
The given system will not have a unique solution when A is not full rank. A will become a rank 1 matrix when the rows are linearly dependent on each other. This condition is satisfied when $p = 6$. Therefore, the system will have a unique solution when $p \neq 6$.

20. A fair coin is thrown repeatedly. What is the probability that on the n^{th} throw, exactly two heads appeared altogether to date. $\binom{n}{k} := \frac{n!}{k!(n-k)!}$ **2 points**

- ☐ $\frac{1}{2^n}$.
- ■ $\binom{n}{2}\frac{1}{2^n}$.
- ☐ $\binom{n}{\frac{n}{2}}\frac{1}{2^n}$.
- ☐ $\binom{n}{1}\frac{1}{2^n}$.

Solution:

There are n trials. We have to consider the set of events with 2 heads and $n-2$ tails. There are $\binom{n}{2}$ sequences containing 2 heads and $n-2$ tails. Each sequence has probability 2^{-n}. Therefore,

$$\mathbb{P}(\text{exactly two heads})=\binom{n}{2}2^{-n}.$$

21. True or False: For three sets A, B and C: $A\setminus(B\cap C) = (A\setminus B)\cap(A\setminus C)$. Here, $A\setminus B$ is defined as $A\cap B^c$. **False** **2 points**

Solution:

L.H.S $= A\setminus(B\cap C)$

$$\begin{aligned} A\setminus(B\cap C) &= A\cap(B\cap C)^c \\ &= A\cap(B^c\cup C^c) \\ &= (A\cap B^c)\cup(A\cap C^c) \\ &= (A\setminus B)\cup(A\setminus C) \\ &\neq \text{R.H.S} \end{aligned}$$

Answer is False.

22. Let A and B be events with probabilities $\mathbb{P}(A)=\frac{3}{4}$ and $\mathbb{P}(B)=\frac{1}{3}$. $\mathbb{P}(A\cap B)$ is bounded by, **3 points**

- ☐ $\frac{1}{12}\leq \mathbb{P}(A\cap B)\leq \frac{1}{4}$.
- ■ $\frac{1}{12}\leq \mathbb{P}(A\cap B)\leq \frac{1}{3}$.
- ☐ $\frac{1}{8}\leq \mathbb{P}(A\cap B)\leq \frac{1}{3}$.
- ☐ $\frac{1}{4}\leq \mathbb{P}(A\cap B)\leq 1$.

Solution:

We have,

$$\mathbb{P}(A\cap B)=\mathbb{P}(A)+\mathbb{P}(B)-\mathbb{P}(A\cup B)\geq \mathbb{P}(A)+\mathbb{P}(B)-1=\frac{1}{12}$$

Also,

$$A\cap B\subseteq A \text{ and } A\cap B\subseteq B.$$

This indicates

$$\mathbb{P}(A\cap B)\leq \min\left[\mathbb{P}(A),\mathbb{P}(B)\right]=\frac{1}{3}.$$

Thus, the final result is

$$\frac{1}{12}\leq \mathbb{P}(A\cap B)\leq \frac{1}{3}.$$

23. Six cups and saucers come in pairs: there are two cups and saucers which are red, two white, and two with stars on. If the cups are placed randomly onto the saucers (one each), the probability that no cup is upon a saucer of the same pattern is: **3 points**

☐ $\frac{1}{4}$.
☐ $\frac{1}{6}$.
■ $\frac{1}{9}$.
☐ $\frac{8}{9}$.

Solution:

Let red be R, white be W and stars be S. Lay out the **saucers** in the order $RRWWSS$. These can be arranged in 6! ways. But since each pair of a given color may be switched without changing appearance, then number of distinct arrangements are

$$\frac{6!}{(2!)^3} = 90.$$

We assume all the combinations are equally likely. The acceptable arrangements of **cups** so that no cup is upon a saucer of the same pattern are $WWSSRR$, $SSRRWW$ and the remaining arrangement in which the first pair of cups is either SW or WS, the second pair is either RS or SR, and the third is either RW or WR. Thus, in total there are 10 combinations. Hence, the required probability is,

$$\begin{aligned} \mathbb{P}(\text{no cup is upon a saucer of the same pattern}) &= \frac{10}{90} \\ &= \frac{1}{9}. \end{aligned}$$

24. True or False: If A and B be independent events, then A^c and B^c are also independent. **2 points**
True

Solution:

In order to check A^c and B^c are independent we have to show, $\mathbb{P}(A^c \cap B^c) = \mathbb{P}(A^c)\mathbb{P}(B^c)$. We are given that A and B is independent. At first, lets check the independence of the events A and B^c.

$$\begin{aligned} \mathbb{P}(A \cap B^c) &= \mathbb{P}\left((A)\backslash(A \cap B)\right) \\ &= \mathbb{P}(A) - \mathbb{P}(A \cap B) \\ &= \mathbb{P}(A) - \mathbb{P}(A)\mathbb{P}(B) \text{ [since } A \text{ and } B \text{ are independent]} \\ &= \mathbb{P}(A)(1 - \mathbb{P}(B)) \\ &= \mathbb{P}(A)\mathbb{P}(B^c) \qquad (1) \end{aligned}$$

Hence, A and B^c are independent. Now,

$$\begin{aligned} \mathbb{P}(A^c \cap B^c) &= \mathbb{P}(B^c\backslash(A \cap B^c)) \\ &= \mathbb{P}(B^c) - \mathbb{P}(A \cap B^c) \\ &= \mathbb{P}(B^c) - \mathbb{P}(A)\mathbb{P}(B^c) \text{ [from equation (1)]} \\ &= (1 - \mathbb{P}(A))\mathbb{P}(B^c) \\ &= \mathbb{P}(A^c)\mathbb{P}(B^c) \end{aligned}$$

Hence, A^c and B^c are independent and the answer is True.

25. Four fair coins are flipped. Given that at least three of the outcomes are alike, what is the probability that all four outcomes are alike? **3 points**

- ☐ $\frac{1}{8}$.
- ☐ $\frac{1}{2}$.
- ■ $\frac{1}{5}$.
- ☐ $\frac{1}{10}$.

Solution:

Among $2^4 = 16$ combinations, 6 combinations (HHTT, HTHT, HTTH, THHT, THTH, TTHH) have exactly 2 outcomes alike and 10 combinations (HHHH, HHHT, HHTH, HTHH, HTTT, THHH, THTT, TTHT, TTTH, TTTT) have at least three outcomes alike. Among these 10 combinations, 2 combinations (HHHH, TTTT) have all four outcomes alike. Therefore, $\mathbb{P}(\text{all alike} \mid \text{at least three alike}) = \frac{2}{10} = \frac{1}{5}$.