Assignment #00

1. Let $a(n) = [-1\ 1\ -1\ 1]$ and $b(n) = [-1\ 0\ 2\ 3\ 4]$ be two discrete time signals. What is the signal obtained when a(n) is linearly convolved with b(n)? **1 point**

   - ☐ [1 –1 –1 –2 –3 3 –1 4]
   - ☐ [–4 1 –3 3 2 1 1 –1]
   - ☐ [1 0 –2 3 0]
   - ☐ [–8 8 –8 8]

2. Let $a(n) = [-1\ 1\ -1\ 1]$ and $b(n) = [-1\ 0\ 2\ 3\ 4]$ be two discrete time signals. What is the signal obtained when a(n) is circularly convolved with b(n)? **1 point**

   - ☐ [4 –2 3 –2 –3]
   - ☐ [–3 2 –4 3 2]
   - ☐ [1 0 –2 3 0]
   - ☐ [–4 1 –3 3 2 1 1 –1]

3. Let $s(t) = 4\delta(t+4) + 3\delta(3t+2) + 4\delta(4t-1) + 5\delta(t-5)$. Let $y(t)$ be the output response to a system with transfer function $H(\mathrm{f}) = \mathrm{sinc(f)} = \frac{\sin(\pi \mathrm{f})}{\pi \mathrm{f}}$ and input $s(t)$. Compute $\int_{-\infty}^{\infty} Y(\mathrm{f})\mathrm{df}$, where $Y(\mathrm{f})$ is the Fourier transform of $y(t)$. **3 points**

   - ☐ 0
   - ☐ 4
   - ☐ 7
   - ☐ 16

4. True or False : FIR filters are used widely because IIR filters need infinite memory and hence are hard to realize. ______________ **1 point**

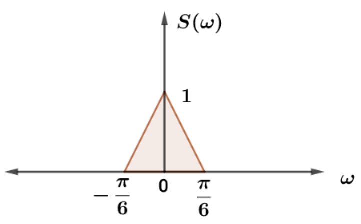
5. Based on the impulse response, choose FIR filters: **2 points**

   - ☐ $H(z) = \frac{1+z^{-1}}{1-z^{-1}}$
   - ☐ $H(z) = \frac{1-z^{-2}}{1+z^{-1}}$
   - ☐ $H(z) = \sum_{i=0}^{25} a_0 z^{-i}$
   - ☐ $H(z) = \frac{1}{z}$

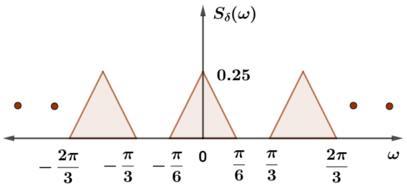
6. What is the DC component of the signal $s(t) = \frac{t}{1+t^2}$? **2 points**

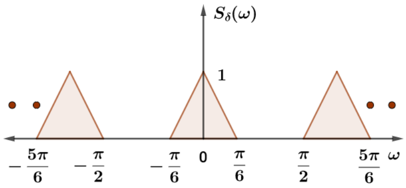
   - ☐ ∞
   - ☐ 0
   - ☐ 2
   - ☐ 1

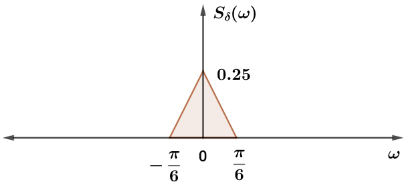
7. Let $s(t)$ be a bandlimited signal given in the figure below: **2 points**

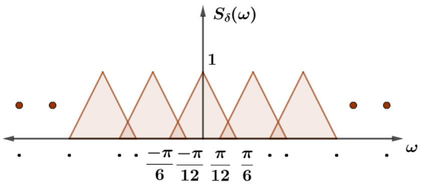


Which of the following figures represent the spectrum $S_\delta(\omega)$ of the signal obtained after sampling $s(t)$ at the rate $f_s = \frac{1}{4}$Hz where $\omega = 2\pi f$:

☐ 

☐ 

☐ 

☐ 

8. Let $H(z) = 1 + a^2z^{-2} + a^4z^{-4} + a^6z^{-6}\cdots$, $|a| < 1$ be the impulse response of a LTI system. Let $X(z) = 1 + az^{-1}$ be the input to the system and let $Y(z)$ be the corresponding output signal. What are the poles and zeros of the signal $Y(z)$? **1 point**

   - ☐ Poles: {a, -a}, Zeros :{-a}
   - ☐ Poles: {-a}, Zeros :{}
   - ☐ Poles: {a}, Zeros :{}
   - ☐ Poles: {a}, Zeros :{-a}

9. Let $Y(z) = a + z$ be the output of a LTI system when the input is $X(z) = 1 - az^{-1}$. True/False: The LTI system is causal. ______________ **1 point**

10. Let $Y(z) = a + z$ be the output of a LTI system when the input is $X(z) = 1 - az^{-1}$. What are the conditions under which the LTI system is BIBO stable? **1 point**

   - ☐ Always
   - ☐ $|a| < 1$
   - ☐ $|a| > 1$
   - ☐ $|a| < 0.5$

11. Let $Y(z) = a + z$ be the output of a LTI system when the input is $X(z) = 1 - az^{-1}$. What is the impulse response of the system under the condition that the system is BIBO stable? **1 point**

   - ☐ $z + 2a + 2a^2z^{-1} + 2a^3z^{-2} + 2a^4z^{-3}\cdots$
   - ☐ $z - 2a + 2a^2z^{-1} - 2a^3z^{-2} + 2a^4z^{-3}\cdots$
   - ☐ $z + (1-a) + (a-a^2)z^{-1} + (a^2-a^3)z^{-2} + (a^3-a^4)z^{-3}\cdots$
   - ☐ $z + (1+a) + (a+a^2)z^{-1} + (a^2+a^3)z^{-2} + (a^3+a^4)z^{-3}\cdots$

12. If 1, ω, ω^2, $\cdots$, ω^{N-1} are the N^{th} roots of unity, what is the value of summation $\sum_{i=0}^{N-1} \omega^{2i}$? **2 points**

- ☐ 0 when N > 2; N otherwise.
- ☐ 2 when N is even; 0 when N is odd.
- ☐ 2 when N is even; N when N is odd.
- ☐ 0 always.

13. What is the energy in the signal $x(t) = \frac{\sin(2\pi t)}{\pi t}$? (Hint: Parseval's theorem) ______________ **2 points**

14. Consider the following matrix **3 points**

$$A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$$

If the eigenvalues of A are 4 and 8, then

- ☐ $x = -4,\ y = 10$
- ☐ $x = 5,\ y = 8$
- ☐ $x = -3,\ y = 9$
- ☐ $x = 4,\ y = 10$

15. Let $\mathbf{x} = [1, 2, \ldots, 100]^{\mathrm{T}}$ be a column vector of size 100×1. Then the rank of the matrix $\mathbf{x}\mathbf{x}^{\mathbf{T}}$ is **3 points**

- ☐ 99
- ☐ 100
- ☐ 1
- ☐ 0

16. The determinant of the matrix $\begin{bmatrix} -1 & 3 & 2 & 8 \\ 0 & 2 & 8 & 1 \\ 0 & 0 & 6 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ is **2 points**

- ☐ 11
- ☐ 48
- ☐ −48
- ☐ 0

17. Let A and B be two matrices of size $n \times n$ such that $\det(AB) = 0$ and $\det(A) \neq 0$, then **3 points**

- ☐ all the eigenvalues of B must be zero.
- ☐ none of the eigenvalues of B must be zero.
- ☐ at least one of the eigenvalues of B must be zero.
- ☐ B should be a zero matrix.

18. If A is a real square matrix, then AA^{T} is **2 points**

- ☐ unsymmetric
- ☐ sometimes symmetric
- ☐ always symmetric
- ☐ skew-symmetric

19. Let $A = \begin{bmatrix} 5 & 3 \\ 10 & p \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$. $Ax = b$ has a unique solution when $p \neq$ ______________. **2 points**

20. A fair coin is thrown repeatedly. What is the probability that on the n^{th} throw, exactly two heads appeared altogether to date. $\binom{n}{k} := \frac{n!}{k!(n-k)!}$ **2 points**

- ☐ $\dfrac{1}{2^n}$.
- ☐ $\dbinom{n}{2}\dfrac{1}{2^n}$.
- ☐ $\dbinom{n}{\frac{n}{2}}\dfrac{1}{2^n}$.
- ☐ $\dbinom{n}{1}\dfrac{1}{2^n}$.

21. True or False: For three sets A, B and C: $A\setminus(B\cap C) = (A\setminus B)\cap(A\setminus C)$. Here, $A\setminus B$ is defined as $A\cap B^c$. ______________ **2 points**

22. Let A and B be events with probabilities $\mathbb{P}(A) = \frac{3}{4}$ and $\mathbb{P}(B) = \frac{1}{3}$. $\mathbb{P}(A\cap B)$ is bounded by, **3 points**

- ☐ $\frac{1}{12} \leq \mathbb{P}(A\cap B) \leq \frac{1}{4}$.
- ☐ $\frac{1}{12} \leq \mathbb{P}(A\cap B) \leq \frac{1}{3}$.
- ☐ $\frac{1}{8} \leq \mathbb{P}(A\cap B) \leq \frac{1}{3}$.
- ☐ $\frac{1}{4} \leq \mathbb{P}(A\cap B) \leq 1$.

23. Six cups and saucers come in pairs: there are two cups and saucers which are red, two white, and two with stars on. If the cups are placed randomly onto the saucers (one each), the probability that no cup is upon a saucer of the same pattern is: **3 points**

- ☐ $\frac{1}{4}$.
- ☐ $\frac{1}{6}$.
- ☐ $\frac{1}{9}$.
- ☐ $\frac{8}{9}$.

24. True or False: If A and B be independent events, then A^c and B^c are also independent. ______________ **2 points**

25. Four fair coins are flipped. Given that at least three of the outcomes are alike, what is the probability that all four outcomes are alike? **3 points**

- ☐ $\frac{1}{8}$.
- ☐ $\frac{1}{2}$.
- ☐ $\frac{1}{5}$.
- ☐ $\frac{1}{10}$.