Assignment #01

1. (True/False) A causal LTI system cascaded with a non-causal LTI system is always non-causal. **1 point**

   ____

2. (True/False) Ideal delay system defined as $y[n] = T(x[n]) = x[n-k]$ is linear. ______________ **1 point**

3. (True/False) A discrete LTI system has an impulse response $h[n]$. If the input $x[n]$ is periodic with period N, i.e. $x[n+N] = x[n]$, the output is also periodic. ______________ **1.5 points**

4. Consider system A that performs the first order difference operation $y[n] = x[n] - x[n-1] = \nabla x[n]$ on the input. System B with impulse response $h[n]$ when cascaded with the first order difference system can recover the input. What is $h[n]$? **1.5 points**

   - ☐ $h[n] = \delta[n]$
   - ☐ $h[n] = u[n]$
   - ☐ $h[n] = n \times u[n]$
   - ☐ $h[n] = 2^n \times u[n]$

5. A discrete LTI system takes an input $x[n]$ and yields $y[n]$. The impulse response of the system is given by $h[n]$. What is the necessary condition on $h[n]$ so that $\max|x[n]| > \max|y[n]|$ is satisfied for all $x[n]$? **2 points**

   - ☐ $\sum_{k=-\infty}^{\infty} |h[k]| = 1$
   - ☐ $\sum_{k=-\infty}^{\infty} |h[k]| \geq 1$
   - ☐ $\sum_{k=-\infty}^{\infty} |h[k]| = 0$
   - ☐ $\sum_{k=-\infty}^{\infty} |h[k]| \leq 1$

6. The following data is measured from a third-order system: **3 points**

$$y = [0.3200, 0.2500, 0.1000, -0.0222, 0.0006, -0.0012, 0.0005, -0.0001]$$

   Assume that the first time index is 0, so that $y[0] = 0.32$. The explicit form of the system is given by

   - ☐ $y[n] = 0.5046 \times (-0.2964)^n - (0.0923 + j1.0855) \times (0.0605 + j0.1892)^n + (-0.0923 + j1.0855) \times (0.0605 - j0.1892)^n$
   - ☐ $y[n] = 0.5046 \times (-0.2964)^n + (0.0923 + j1.0855) \times (0.0605 + j0.1892)^n + (-0.0923 + j1.0855) \times (0.0605 - j0.1892)^n$
   - ☐ $y[n] = 0.5046 \times (-0.2964)^n - (0.0923 + j1.0855) \times (0.0605 + j0.1892)^n + (0.0923 - j1.0855) \times (0.0605 - j0.1892)^n$
   - ☐ $y[n] = 0.5046 \times (-0.2964)^n - (0.0923 + j1.0855) \times (-0.0605 + j0.1892)^n + (-0.0923 + j1.0855) \times (0.0605 + j0.1892)^n$

7. Let the state space representation of a LTI system has $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. Which among these can also represent the same system? **1 point**

- ☐ $\mathbf{A} = \begin{bmatrix} 10 & 12 \\ 12 & 10 \end{bmatrix}$
- ☐ $\mathbf{A} = \begin{bmatrix} 13 & -15 \\ 16 & -22 \end{bmatrix}$
- ☐ $\mathbf{A} = \begin{bmatrix} 12 & -17 \\ 19 & 5 \end{bmatrix}$
- ☐ $\mathbf{A} = \begin{bmatrix} 27 & -42 \\ 16 & -25 \end{bmatrix}$

8. Let an LTI system be described by **1 point**

$$\mathbf{x}[n+1] = \begin{bmatrix} 1 & 0 \\ 1 & -2 \end{bmatrix} \mathbf{x}[n] + \begin{bmatrix} 2 \\ -1 \end{bmatrix} f[n]; \quad y[n] = \begin{bmatrix} -1 & 3 \end{bmatrix} \mathbf{x}[n].$$

What is the transfer function $H(z) = \frac{Y(z)}{F(z)}$ of the system?

- ☐ $\frac{-5}{z+2}$
- ☐ $\frac{-2}{z-1}$
- ☐ $\frac{-2}{(z-1)(z+2)}$
- ☐ $\frac{-1}{(z-1)(z+2)}$

9. Let an auto-regressive system with output $y[n]$ for the forcing function $f[n]$ be given by $y[n+2] - 3y[n+1] + 2y[n] = 2f[n]$. Which among the following gives a state-space representation of the system? **2 points**

- ☐ $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}, \quad d = 0$
- ☐ $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}, \quad d = 0$
- ☐ $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}, \quad d = 0$
- ☐ $\mathbf{A} = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 0.5 \\ -0.5 \\ 0 \end{bmatrix}, \quad d = 0$

10. A LTI system with forcing function $f[n]$ and output $y[n]$ is represented using two state variables $u[n]$ and $w[n]$ as follows: **1.5 points**

$$u[n+1] = 2w[n] + f[n], \quad w[n+1] = 3u[n] + 2f[n], \quad y[n] = 4u[n] + 5w[n] - f[n].$$

What is the transfer function of the LTI system?

- ☐ $\frac{6z^{-2} - z^{-1} + 1}{1 - 6z^{-2}}$
- ☐ $\frac{5z^{-2} - 1}{1 - 6z^{-2}}$
- ☐ $\frac{23z^{-2} + 10z^{-1} + 2}{1 - 6z^{-2}}$
- ☐ $\frac{37z^{-2} + 14z^{-1} - 1}{1 - 6z^{-2}}$

11. Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be two arbitrary vector spaces. Then, which among the following are vector spaces? **2 points**

- ☐ $\mathcal{S}_1 \cup \mathcal{S}_2$

☐ $\mathcal{S}_1 \cap \mathcal{S}_2$

☐ $\{(\mathbf{v}_1, \mathbf{v}_2) \mid \mathbf{v}_1 \in \mathcal{S}_1, \mathbf{v}_2 \in \mathcal{S}_2\}$ with $(\mathbf{u}_1, \mathbf{u}_2) + (\mathbf{v}_1, \mathbf{v}_2) = (\mathbf{u}_1 + \mathbf{v}_1, \mathbf{u}_2 + \mathbf{v}_2)$

☐ $\mathcal{S}_1 \setminus \mathcal{S}_2$

12. Let $\mathcal{A} = \{\omega, \omega^2, \cdots, \omega^N\}$ be N^{th} roots of unity, where $\omega = e^{j\frac{2\pi}{N}}$. We define addition operation as $\mathbf{v}_1 \boxplus \mathbf{v}_2 = \mathbf{v}_1\mathbf{v}_2\ \forall \mathbf{v}_1, \mathbf{v}_2 \in \mathcal{A}$. Eg: $\omega^2 \boxplus \omega^3 = \omega^5$. We define scalar multiplication as $a \cdot \mathbf{v} = a\mathbf{v}, \forall \mathbf{v} \in \mathcal{A}$, where the scalar $a \in \mathcal{A}$. What is the additive identity for the defined operations and the sets? **1 point**

☐ 0

☐ 1

☐ Does not exist

☐ Any element in $\mathcal{A}$

13. (True/False) For the sets and operations defined in Problem#12, is $\mathcal{A}$ a vector space? **1.5 points**

____