Assignment #02

1. True/False : $\mathbf{x} = [5\ 2\ 6]^{\mathrm{T}}$ belongs to the vector space spanned by $\mathbf{a} = [1\ 0\ 1]^{\mathrm{T}}$ and $\mathbf{b} = [1\ 1\ 0]^{\mathrm{T}}$. **1 point**

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2. Let V be a vector space in $\mathbb{R}^2$. For what values of c_1 and c_2, the vector $\mathbf{x} = [c_1\ c_2]^{\mathrm{T}}$ is not an element in the basis of V? **1 point**
   - ☐ $c_1 = 0,\ c_2 = 0$
   - ☐ $c_1 = 1,\ c_2 = -1$
   - ☐ $c_1 = 2,\ c_2 = 4$
   - ☐ $c_1 = 0,\ c_2 = 2$

3. The L_2 norm of $f(x) = 3x^2 - 1$ over $[-1, 1]$ is **1 point**
   - ☐ $\frac{8}{5}$
   - ☐ $\sqrt{\frac{8}{5}}$
   - ☐ $\sqrt{\frac{4}{5}}$
   - ☐ $\frac{16}{25}$

4. Let $\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -4 \\ \alpha \\ 2 \end{pmatrix}$. The vectors $\mathbf{u}$ and $\mathbf{v}$ are orthogonal for what value of α? **1 point**

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5. True/False : For any matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$: the inner product $(\mathbf{A}\mathbf{x}, \mathbf{y}) = (\mathbf{x}, \mathbf{A}^{\mathrm{T}}\mathbf{y})$ for every $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{y} \in \mathbb{R}^m$. ______________ **1 point**

6. Let $\mathbf{x} = [-2\ 1\ 2]^{\mathrm{T}}, \mathbf{y} = [-1\ 2\ 3]^{\mathrm{T}}$ and $\mathbf{z} = [-4\ \ -1\ 0]^{\mathrm{T}}$. If $\mathbf{x}, \mathbf{y}$ and $\mathbf{z}$ are linearly dependent such that $\alpha\mathbf{x} + \beta\mathbf{y} + \mathbf{z} = 0$, then $\alpha + 3\beta$ is ______________ **1.5 points**

7. If $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}\}$ is an orthonormal set in $\mathbb{R}^n$, then $\alpha(\mathbf{u_1} + \mathbf{u_2} + \mathbf{u_3} + \mathbf{u_4})$ is a unit vector when α is **2 points**
   - ☐ 1
   - ☐ 1/2
   - ☐ 1/4
   - ☐ -1/2

8. The minimum value of $x - 2y + 2z$ subject to the constraint $x^2 + y^2 + z^2 = 1$ is **2 points**
   - ☐ 9
   - ☐ 3
   - ☐ -3
   - ☐ -1

9. $\cos(\omega t + \phi)$, $\cos(\omega t - \phi)$, $\sin(\omega t + \phi)$ and $\sin(\omega t - \phi)$ belong to the vector space V for some constant ϕ. What can you deduce about the dimension d of the vector space? **2 points**
   - ☐ $d = 2$
   - ☐ $d \geq 2$
   - ☐ $d = 4$
   - ☐ $d \geq 4$

10. Let A and B be subspaces of the vector space of 2×2 matrices defined over integers. Then, subspace C is said to be the direct sum of A and B if $C = A + B$ and $A \cap B = \{0\}$. The basis of the subspaces are given by $\mathcal{B}_A = \left\{ \begin{bmatrix} m-26 & 4 \\ 0 & m \end{bmatrix}, \begin{bmatrix} 0 & m \\ 1 & 2 \end{bmatrix} \right\}$ and $\mathcal{B}_B = \left\{ \begin{bmatrix} 6m & 0 \\ 1 & 1 \end{bmatrix} \right\}$. For what value of m is $C = A + B$ not a direct sum? ____________ **2 points**

11. Let A and B be two vector spaces defined over $\mathbb{R}$. Let A be a vector space spanned by $S_A = \{t^3 - 2t^2 + 3t - 2,\ 3t^3 - 5t^2 + 4t - 1,\ 2t^2 - 10t + 10,\ 3t - 5, 6t^2 - 20\}$. Let B be a vector space spanned by $S_B = \{3t^3 - 6t^2 + 9t - 6,\ 2t^3 - 3t^2 + t + 1, t^2 - 2t\}$. Which of the following relationship between the vector spaces hold: **2.5 points**

    - ☐ $A = B$
    - ☐ $A \subset B$
    - ☐ $B \subset A$
    - ☐ There is no relation between them

12. Which of the following sets form a basis for the vector space V having Hamel basis $\mathcal{B} = \{1, 2t^4 + 6t^2 + 5, 3t^2 - 7t + 6\}$? **3 points**

    - ☐ $S_1 = \{-3t^4 - 9t^2, 2t^4 + 14t - 7, 2, 3t^2 - 7t + 6\}$
    - ☐ $S_2 = \{-2.5, -t^4 - 3t^2, t^4 + 7t - 3.5\}$
    - ☐ $S_3 = \{t^4 + 3t^2 + 2.5, t^4 + 7t - 3.5, t^4 + 3t^2\}$
    - ☐ $S_4 = \{3, 5t^4 + 15t^2 + 12.5, 3t^3 - 7t^2 + 6t\}$